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DCU-1603010102010300 Seat No. ______

M. Sc. (Sem. I) (CBCS) (W.E.F. 2016) Examination

August - 2022

Physics : CT - 03

(Quantum Mechanics - I)

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) This question paper contains 10 questions carrying 14 marks each.

(2) The student can write any five questions.

(3) Symbols have their usual meanings.

1 Answer the following questions in brief : **14**

(a) By which equations, the dimensionless coordinates ; 'ξ' and '$\in$' are defined?

(b) How the operators ; a and $a^{\dagger}$ are defined?

(c) Prove : $\left[J_z, J_-\right] = -\hbar J_-$.

(d) Show that $\left[J_+, J_-\right] = 2\hbar J_z$.

(e) Give the relations of Rectangular and Spherical Polar Coordinates.

(f) How the Hamiltonian, H can be represented in terms of a and $a^{\dagger}$?

(g) Prove that $J_+J_- = \vec{J}^2 - J_z^2 + \hbar J_z$.

2 Answer the following questions in brief : 14

(a) What is trial wave function? How it is selected?

(b) Write the significance of Perturbation Theory in Quantum Mechanics.

(c) Define time dependent perturbation in terms of unperturbed Hamiltonian (Ho) and perturbed Hamiltonian $(H(t))$. Write the main application of this time dependent perturbation theory.

(d) Write the full form of WKB in the WKB approximation. Why it is called the semi-classical approximation?

(e) Write the three dimensional Schrödinger equation in Cartesian coordinates.

(f) In the first order time independent perturbation theory, the following equation is available,

$$(E_k - E_m)c_k^{(1)} + H'_{km} + W^{(1)}\delta_{km} = 0$$

Prove that for $k = m$, it given $H'_{kk} = W^{(1)}$.

(g) In three dimensional Schrödinger equation in polar coordinates, by which method the angular and radial parts of the equations are separated? Explain in brief.

3 Answer the following questions in detail : 14

(a) In the exercise of finding a solution of the Schrödinger equation for the Linear Harmonic Oscillator derive up to the below given equation $\frac{d^2h}{d\xi^2} - 2\xi\frac{dh}{d\xi} + h(\in -1) = 0$.

(b) Depict the classical and quantum probabilities for $n = 0$ and $n = 1$, discuss both the cases in detail.

4 Answer the following questions in detail : **14**

(a) What is 'Coordinate Transformation'? Obtain $\vec{L}_z$ in (θ, Φ) - representation as $\vec{L}_z = -i\hbar \frac{\partial}{\partial \phi}$.

(b) Using the concept of Raising and Lowering operators for angular momentum, obtain the following relations -

(i) $\lambda_J - m_{\max}\left(m_{\max} + 1\right)\hbar^2 = 0$

(ii) $\lambda_J - m_{\min}\left(m_{\min} - 1\right)\hbar^2 = 0$

5 Answer the following questions in detail : **14**

(a) Discuss the Harmonic Oscillator Energy Spectrum for one dimensional harmonic oscillator.

(b) Solve the following equation for One Dimensional Harmonic Oscillator using power series method,

$$\frac{d^2h}{d\xi^2} - 2\xi \frac{dh}{d\xi} + h(\in -1) = 0$$

6 Write short notes on : **14**

(a) The Raising, Lowering and Number Operators.

(b) Spherical Harmonics.

7 Answer the following questions in detail : **14**

(a) Explain variational method in Quantum Mechanics with necessary derivations.

(b) Apply variational method to solve Simple Harmonic Oscillator. Take your trial wave function as $\psi = Ce^{-\lambda x^2}$

8 Answer the following questions in detail : **14**

(a) Discuss the WKB approximation in detail and obtain relations for classically accessible region (for $E > V(x)$) and classically inaccessible region (for $E < V(x)$).

(b) In the time independent perturbation theory for the non-degenerate case, obtain the following equation,

$$\left(E_n - E_m\right)c_k^{(2)} = \sum_n \left(H'_{kn} - W^{(1)}\,\delta_{kn}\right)c_n^{(1)} - W^{(2)}\,\delta_{km} = 0.$$

9 Answer the following questions in detail : **14**

(a) Discuss the second order time independent perturbation theory in detail and derive the following expression,

$$\upsilon(2) = \sum_k \left\{ \sum_n \frac{H'_{kn}}{E_k - E_m} \frac{H'_{nm}}{E_n - E_m} + \frac{H'_{kn} H'_{nm}}{\left(E_k - E_m\right)^2} \right\} u_k$$

(b) For the time dependent peturbation derive the following general formulation using Schrödinger equation,

$$\left[i\hbar \frac{dc_m(t)}{dt} \right] = \sum_n \langle \Phi_m | H_1(t) | \Phi_n \rangle c_n(t) \cdot e^{i(E_m - E_n)t/\hbar}$$

10 Answer the following questions in detail : **14**

(a) The angular form of Schrödinger equation is written as follows :

$$\frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial Y(\theta, \phi)}{\partial\theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2 Y(\theta, \phi)}{\partial\phi^2} + \lambda Y(\theta, \phi) = 0$$

By assuming, $Y(\theta, \phi) = H(\theta)\,\Phi(\phi)$

Convert this angular equation into two different equations: for angular equation $-\theta$ and for angular equation $-\phi$.

(b) Apply radial Schrödinger equation for Coulomb potential, $V(r) = -\frac{C}{r}$ and obtain the energy eigen values of the form, $E_n = -\frac{mZ^2}{2\hbar^2 n^2}$.